Disordered Ultra-cold Gases

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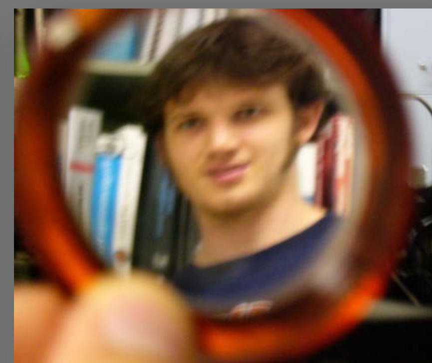
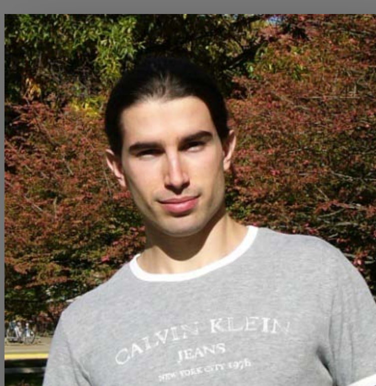

David McKay


Matt Pasienski



Stan Kondov


David Chen


Carrie Meldgin


Will McGehee


Paul Koehring

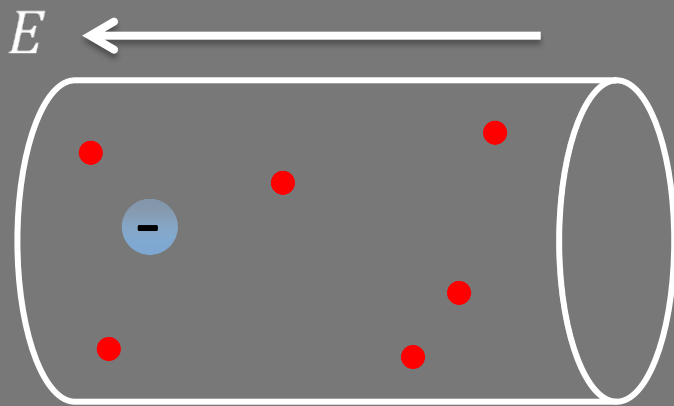
Corrections / Additions

- Easy to create spin-independent honeycomb optical lattices *in principle*
- Holographic lattices
- Failure of LDA / MFT: fluctuations over long length scales near critical point

Today: outline

- Anderson localization
- Disordered Bose-Hubbard model
- Bandmapping / real eigenstates
- Transport / phase slips / superfluidity
  (all superfluids are dissipative)

Anderson localization



Diffusive transport (Drude/Boltzmann)

$$v = -eE\tau/m$$

$$j = nqv = ne^2\tau E/m = \sigma E$$

$$\sigma = ne^2\tau/m$$

$$D \propto \sigma$$

Weak localization

Mean free path $l = v\tau \gg$ deBroglie wavelength (or $1/k_F$)

$$P = |A_1 + A_2 + \cdots|^2$$

$$P_{clas} = 2|A|^2$$

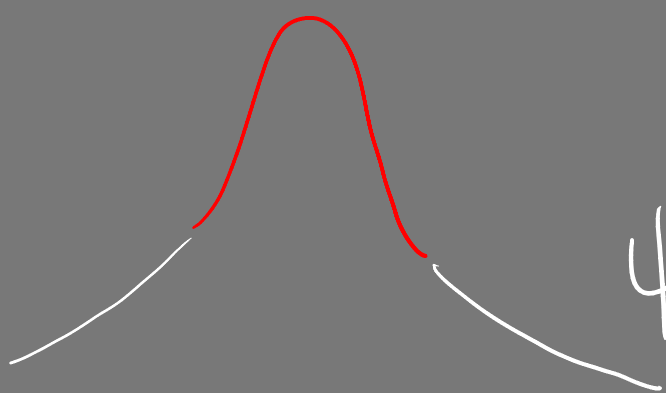
$$P_{QM} = |A_1|^2 + |A_2|^2 + 2A_1A_2 = 4|A|^2$$

Constructive interference for returning to A…σ is reduced

Anderson localization

Anderson:

single-particle states can be completely localized



$\psi \sim e^{-r/L}$

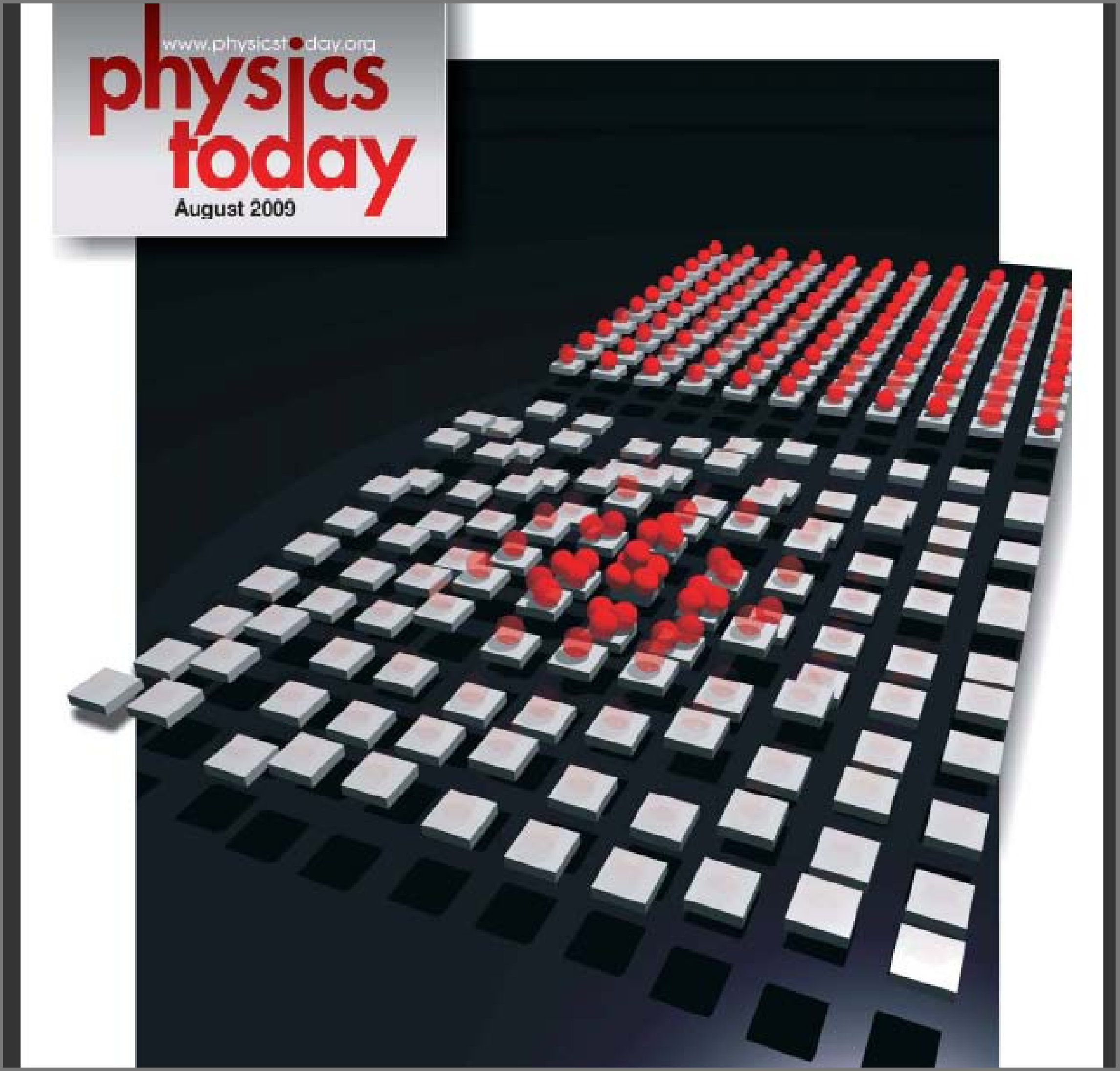
Consequence:

lack of diffusion

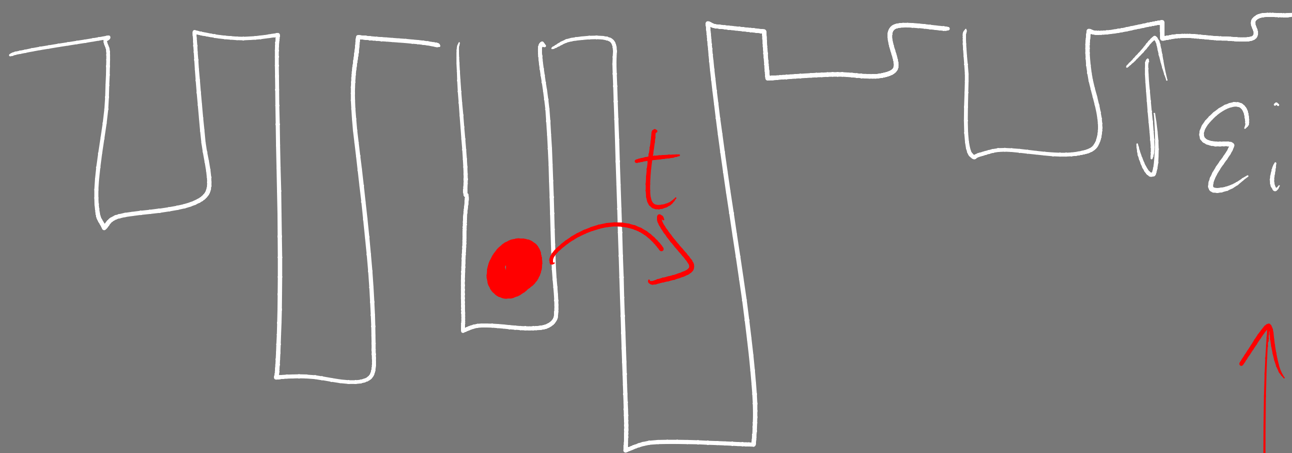
$$D = 0$$

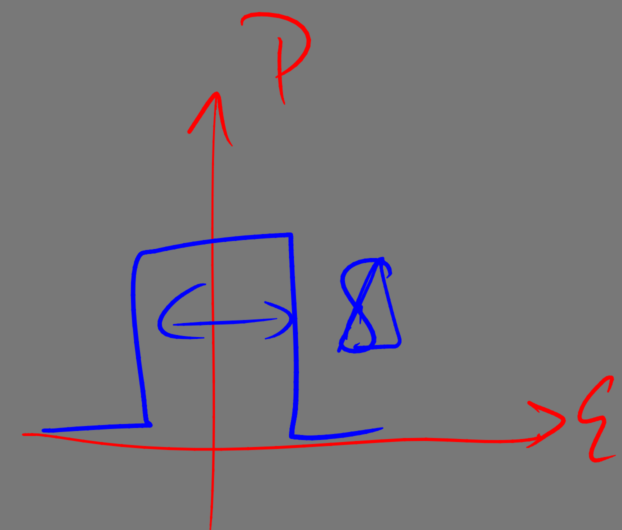
$$\sigma = 0$$

Anderson Localization



Anderson Localization





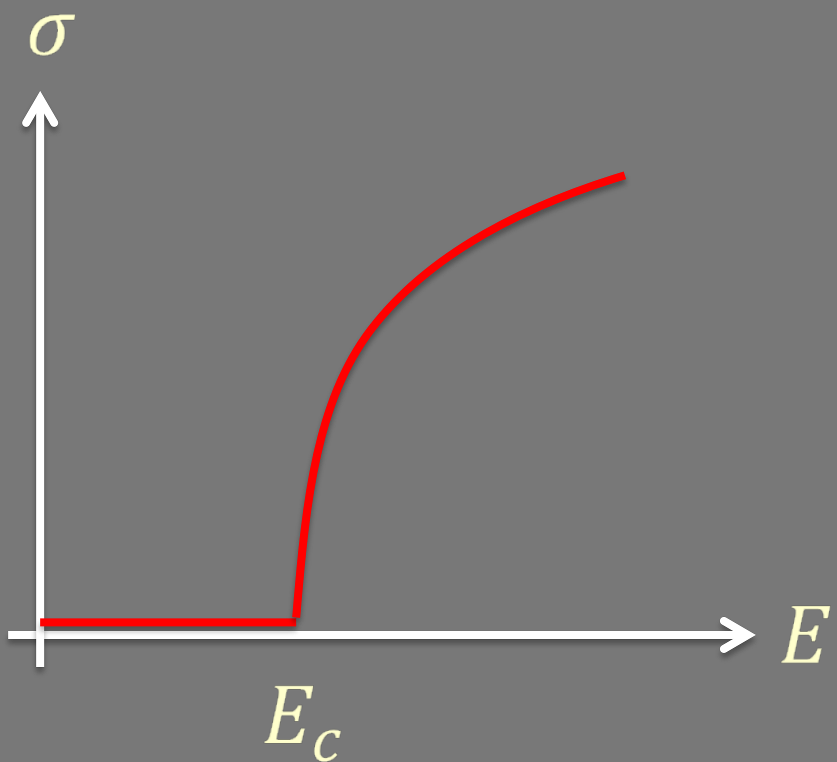
Key features:

- Non-interacting theory:

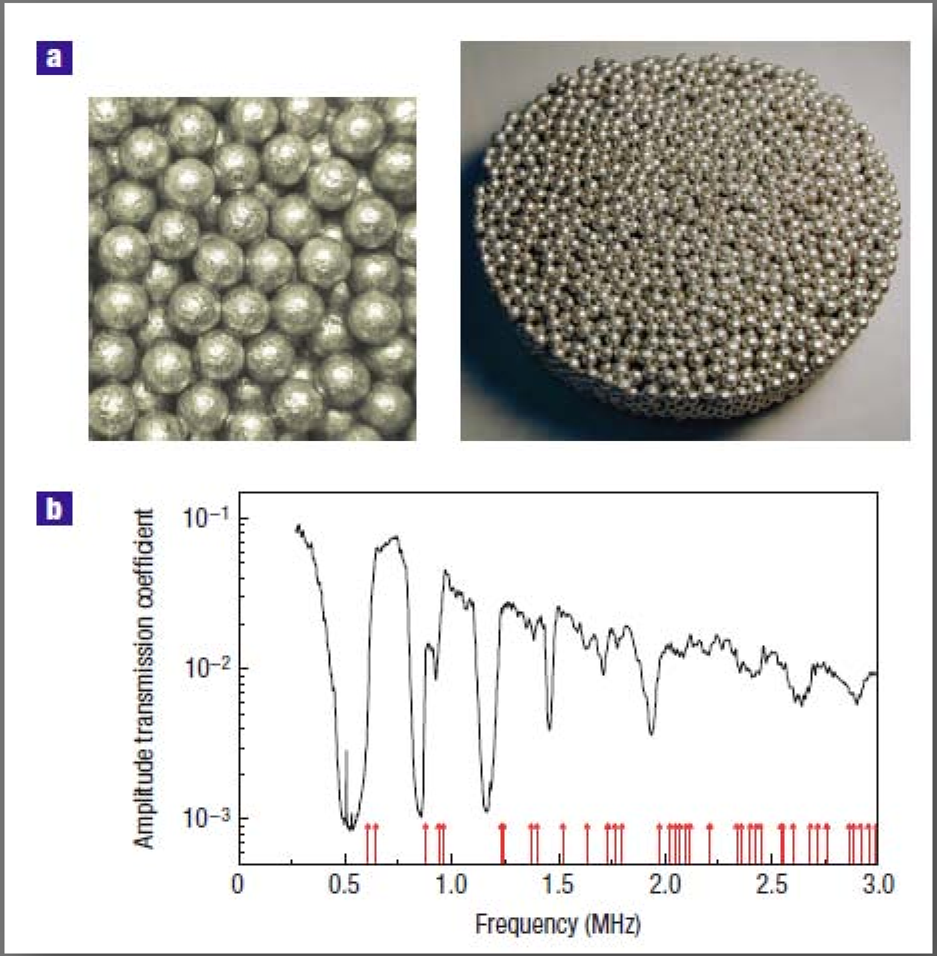
$$H = \sum_i \epsilon_i + t \sum_{\langle ij \rangle} a_i^\dagger a_j$$

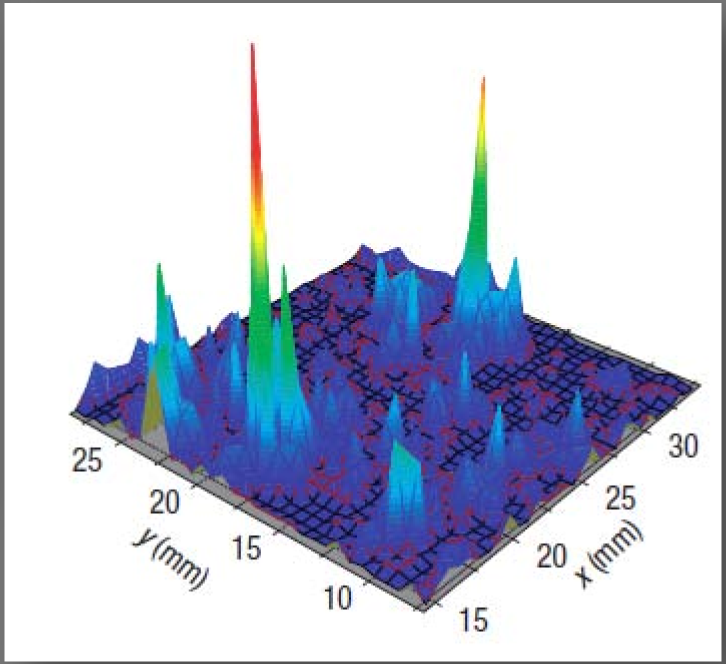
- All states in $d = 1{,}2$ localized for infinitesimal disorder
- Critical value required in $\mathrm{d} = 3$: $\Delta \sim t$ (for energies below mobility edge

Mobility Edge



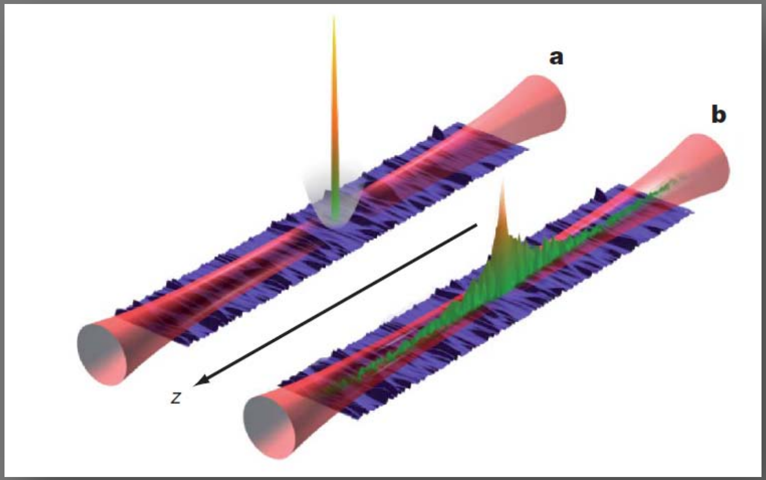
Classical Anderson Localization

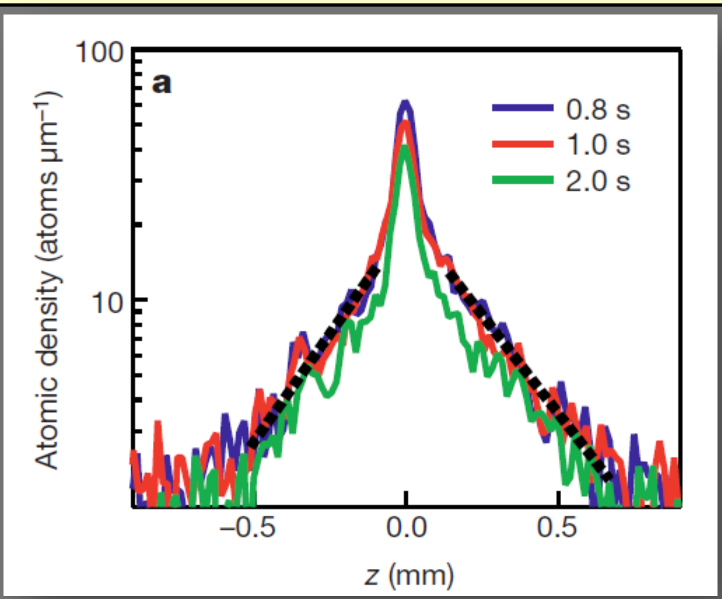


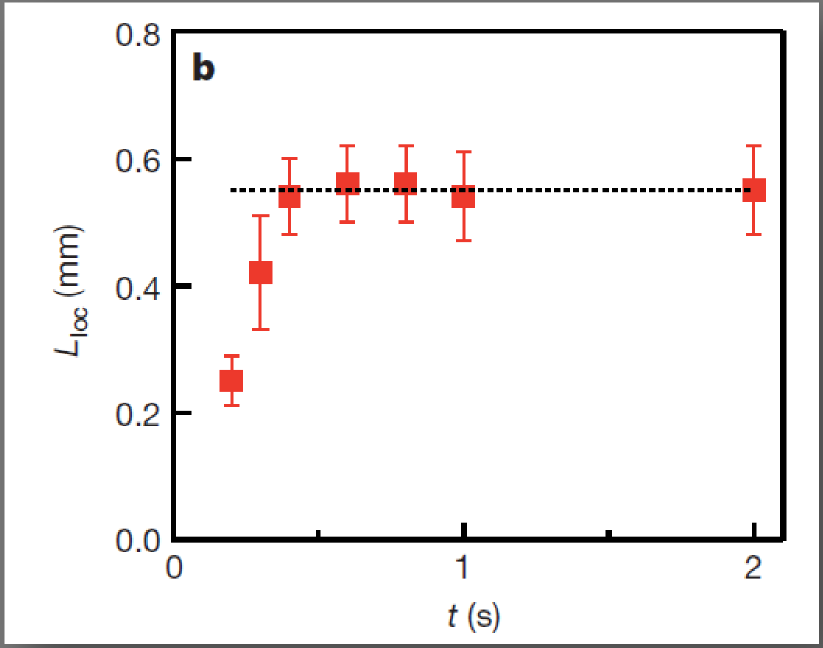


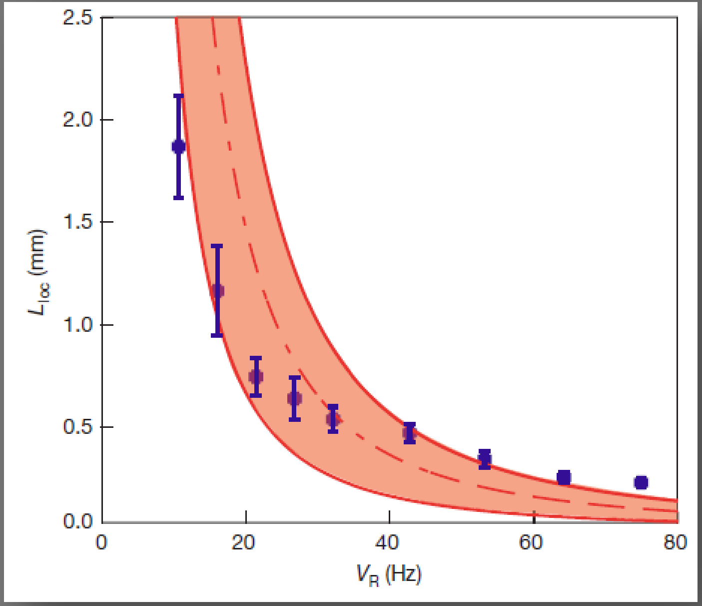
Van Tiggelen, doi:10.1038/nphys1101 (2008)

In cold gases (Aspect, Inguscio)

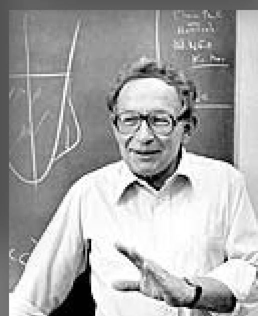








Disorder + Interactions



Interactions

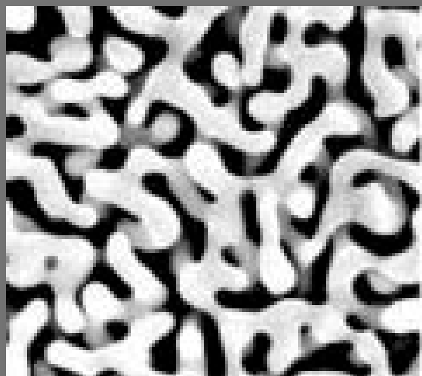
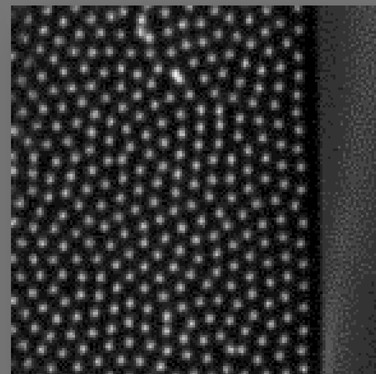
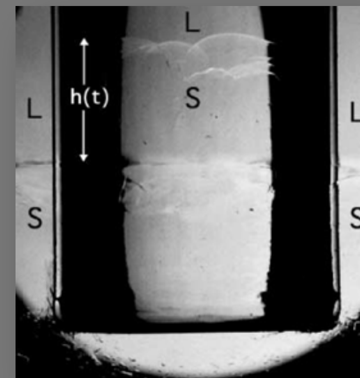
localization ➡ Mott-Insulator

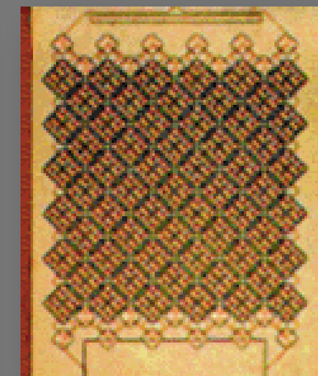
Disorder

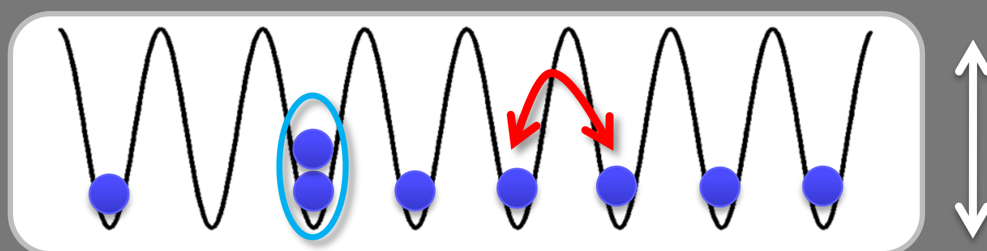
Anderson localization ➡ Insulator

What about the combination?

The Bose-Hubbard model





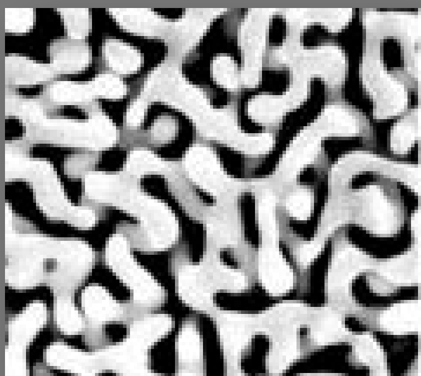
ε: site energies

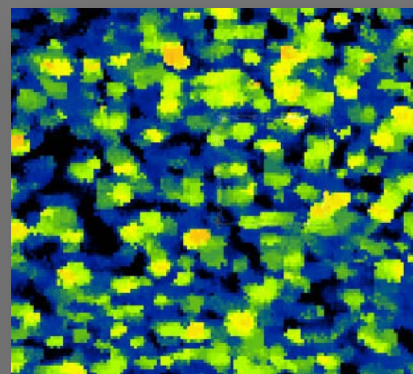
U: interaction energy t: tunneling energy

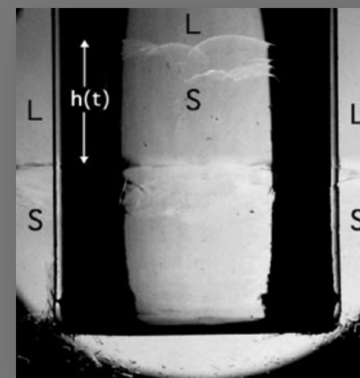
$$H=-t\sum_{\langle ij\rangle}\left(b_i^{\dagger}b_j+b_j^{\dagger}b_i\right)+\frac{U}{2}\sum_i n_i\left(n_i-1\right)$$

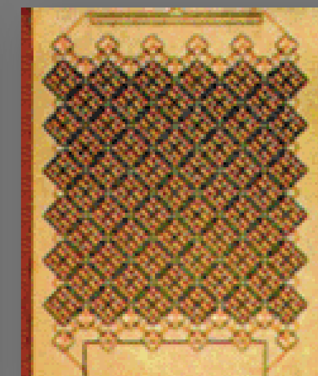
Simulating the Disordered BH model

Disordered bosonic materials

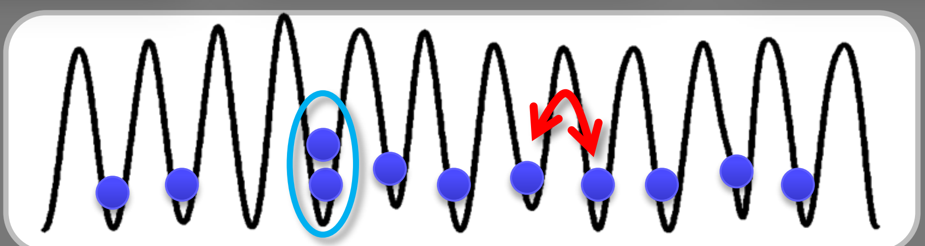








disordered lattice



ε_i: site energies

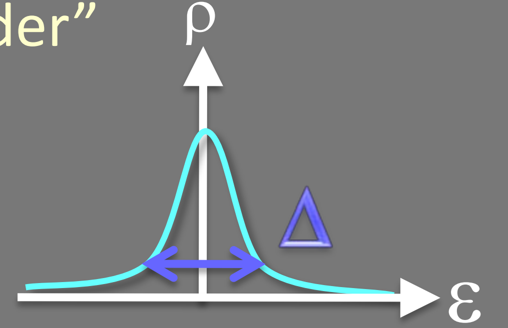
"diagonal disorder"

U_i: interaction energy

t_{ij}: tunneling energy

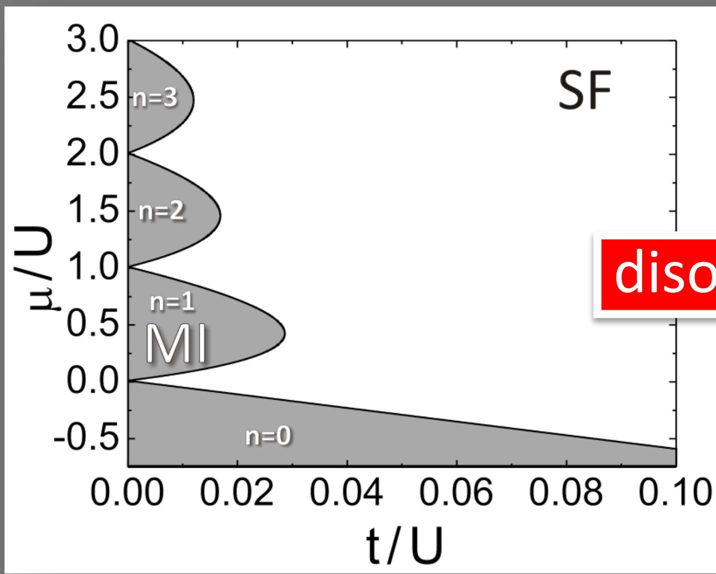
"off-diagonal disorder"

$$H = \sum_i n_i \varepsilon_i - \sum_{\langle ij \rangle} t_{ij} \left(b_i^\dagger b_j + b_j^\dagger b_i \right) + \frac{1}{2} \sum_i U_i n_i \left(n_i - 1 \right)$$



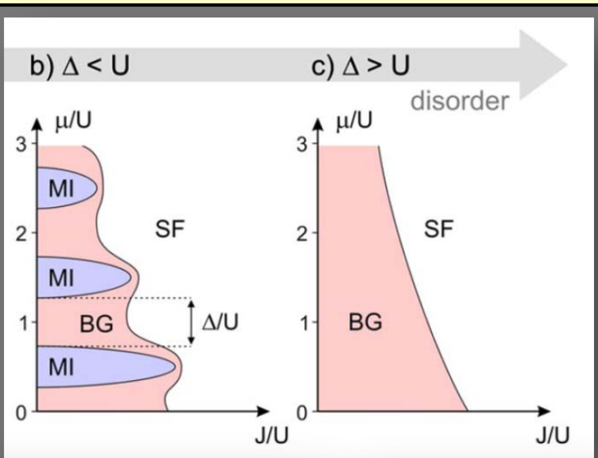
Disordered BH model: theory

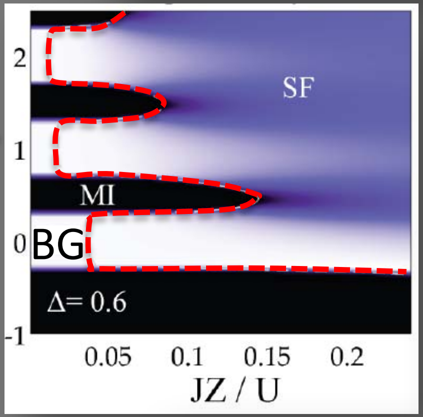
“filling”

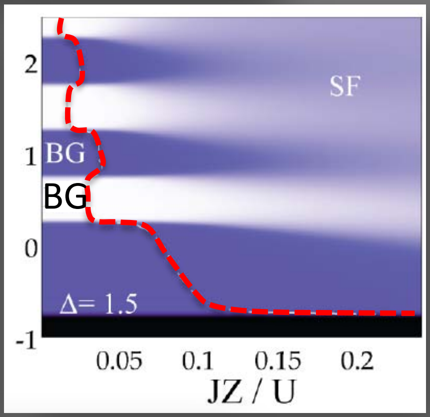


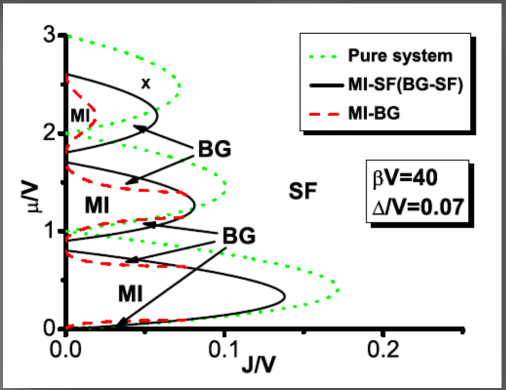
increasing lattice depth

disorder







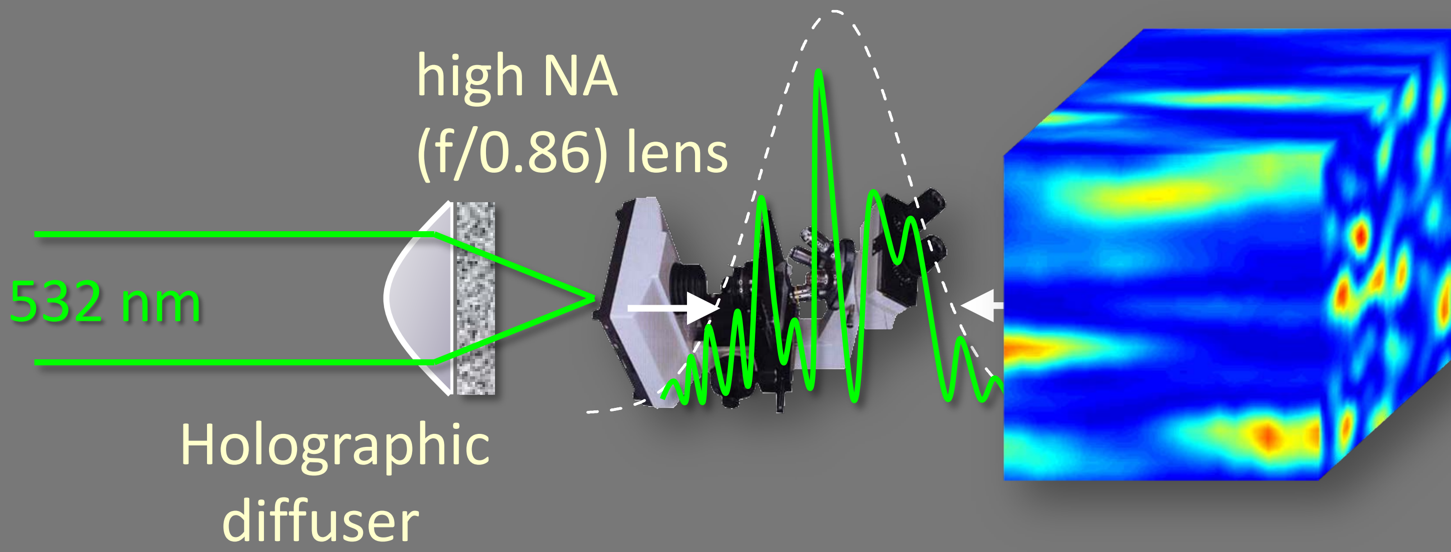


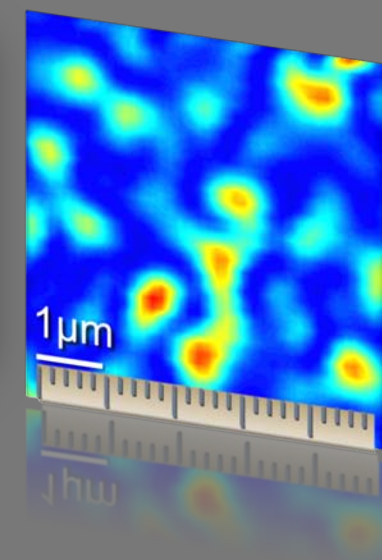
SF: superfluid MI: Mott-insulator

BG: Bose-glass (gapless, compressible IN)

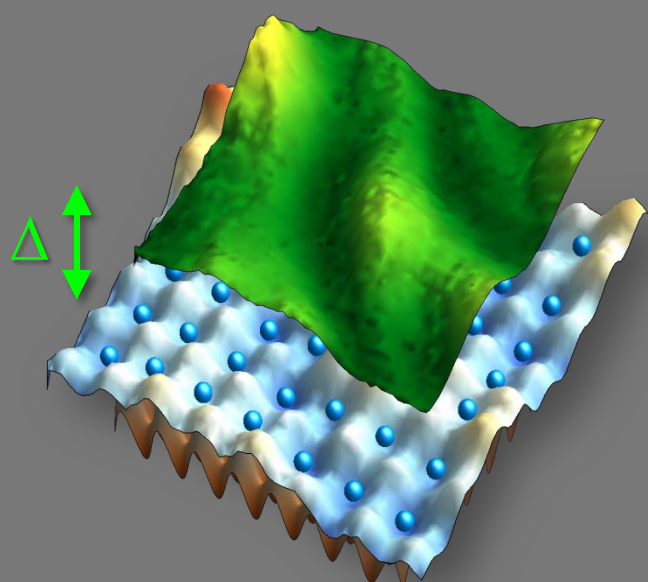
MG: Mott-glass (gapless, incompressible IN)

Controllable disorder: speckle





Known disorder!

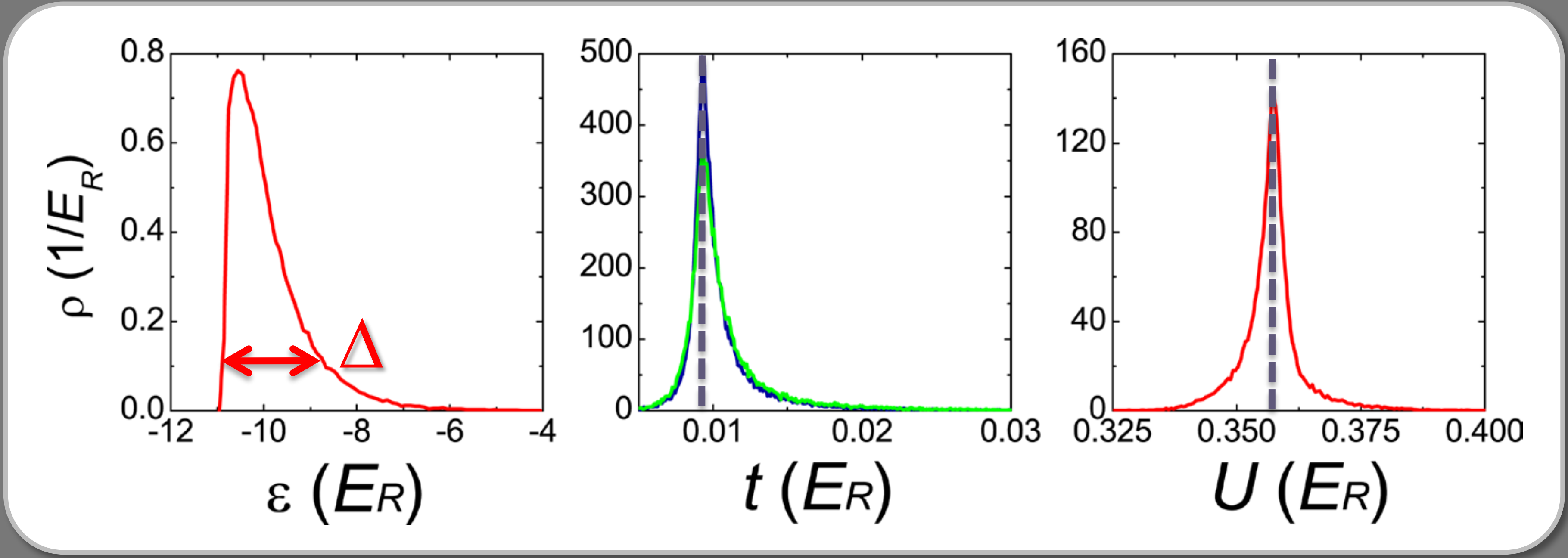


First realization of fine-grained disorder in a lattice

Aspect, Ertmer, Hulet, Inguscio…

BH parameters

Calculation by Ceperley's group using known potential



Bandmapping

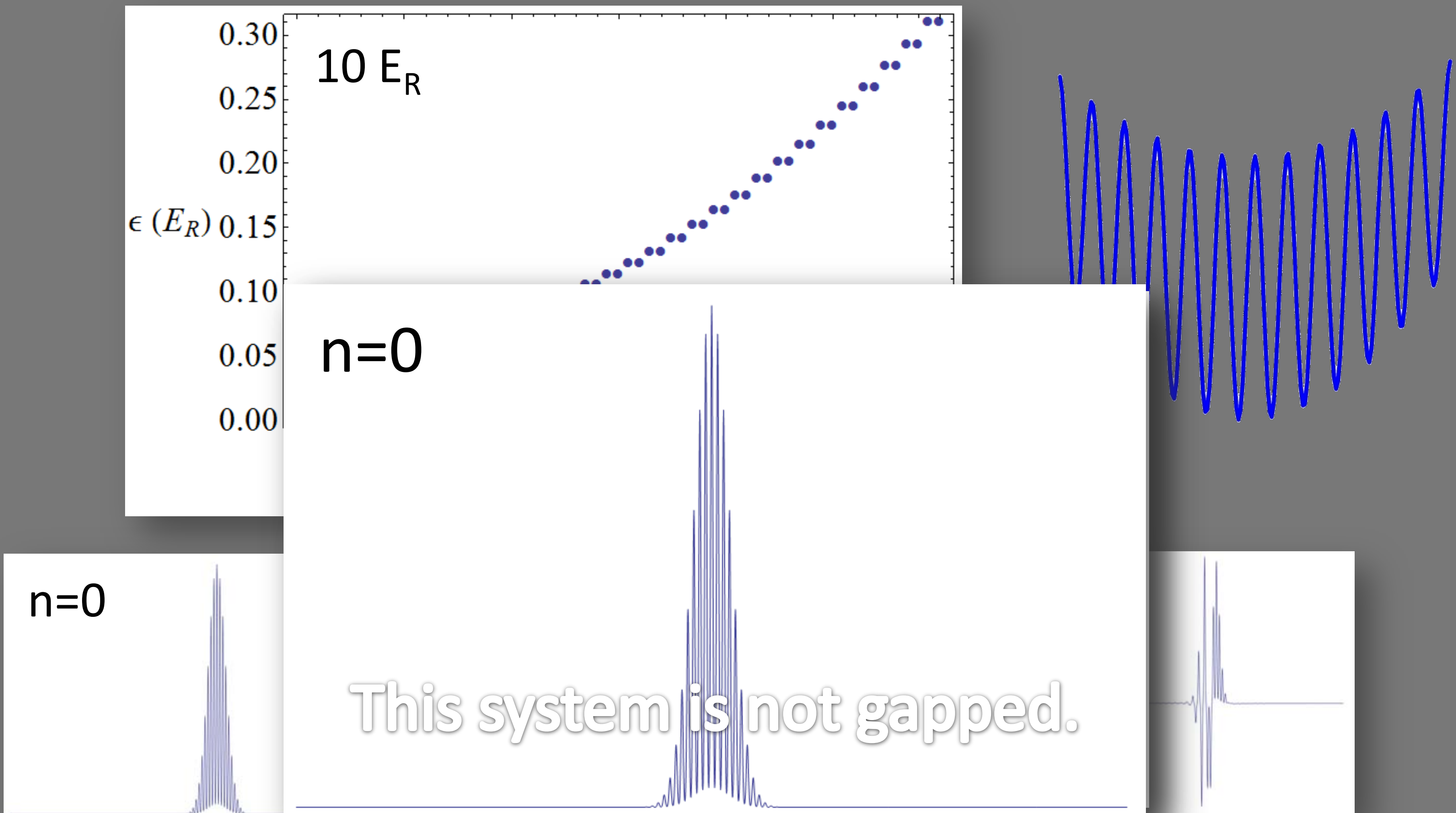
Maps quasi-momentum → momentum

(for weak interactions, low quasi-momentum)

Lattice Thermodynamics for Ultra-cold Atoms, PRA **79** 063605 (2000)

Real single-particle eigenstates

Exactly known (PRA **72**, 033616 (2005); **79**, 063605 (2009))

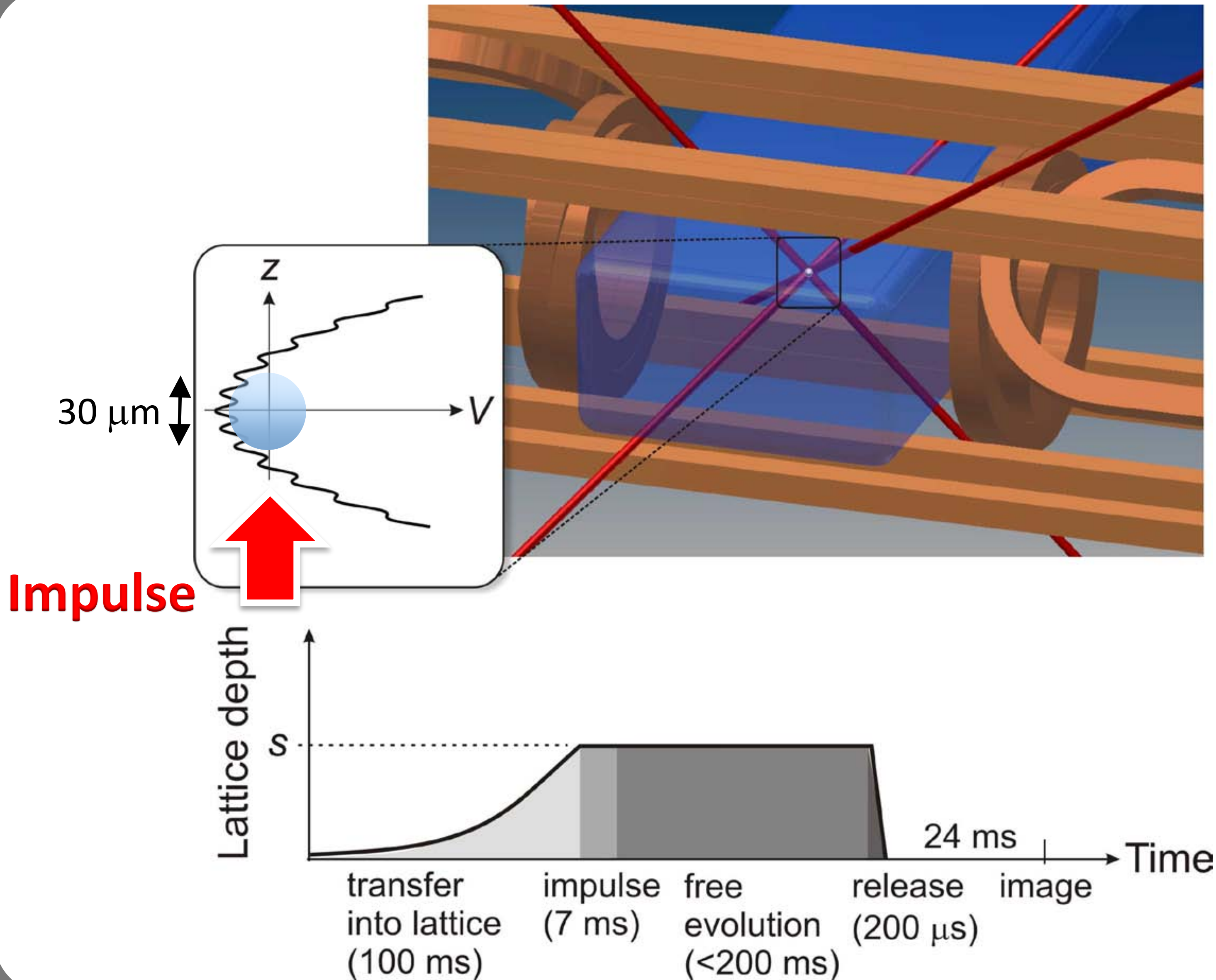


This system is not gapped.

Transport Measurements

resistivity $\rho \propto \frac{V}{I}$

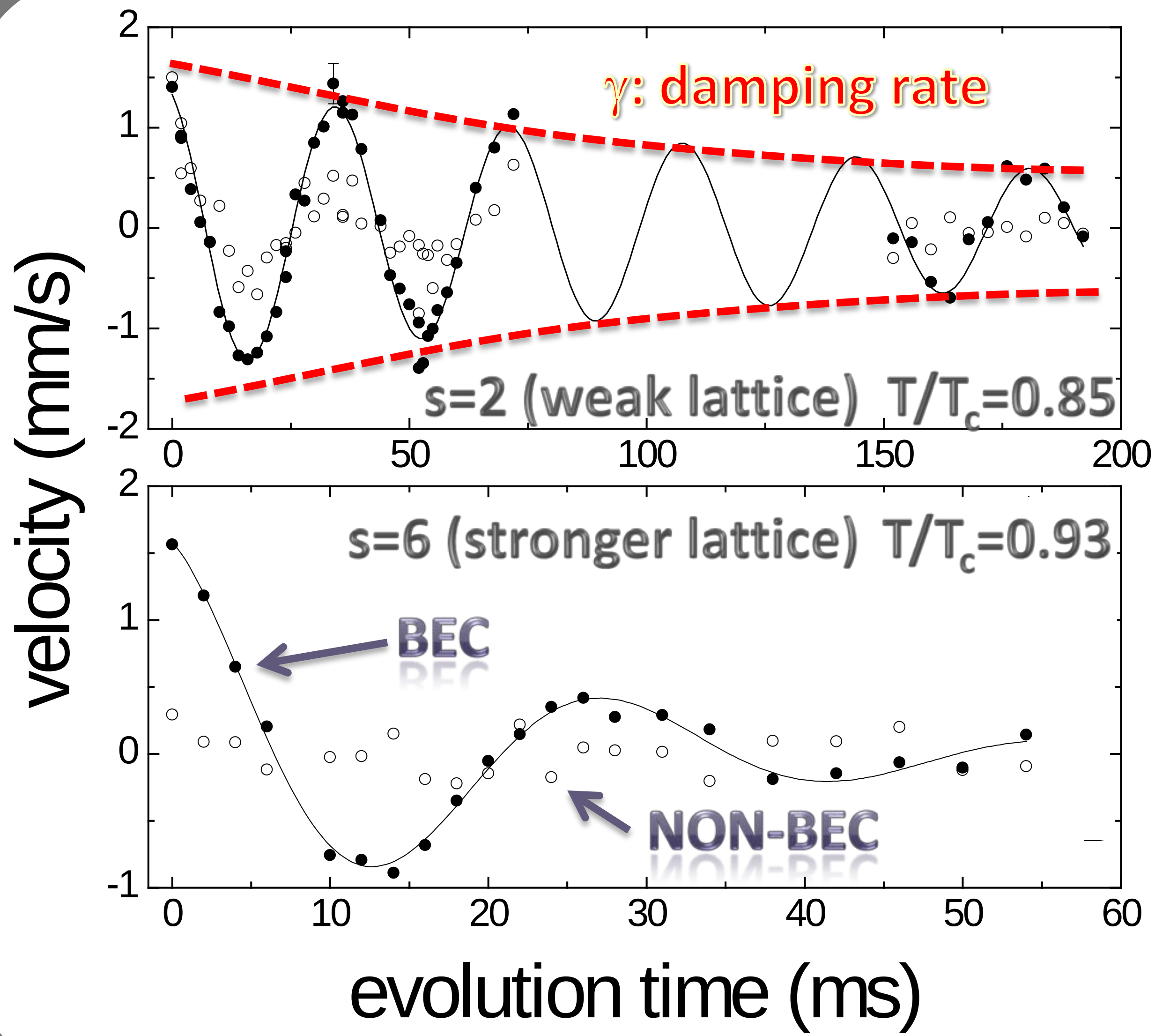
Transport



30 μm
z
V
Impulse
Lattice depth
s
24 ms
Time
transfer into lattice (100 ms)
impulse (7 ms)
free evolution (<200 ms)
release (200 μs)
image

Motion

Observable: center-of-mass velocity



We measure:
- Damping rate (resistance)

We change:
- Lattice depth (t/U)
- Temperature
- Disorder strength

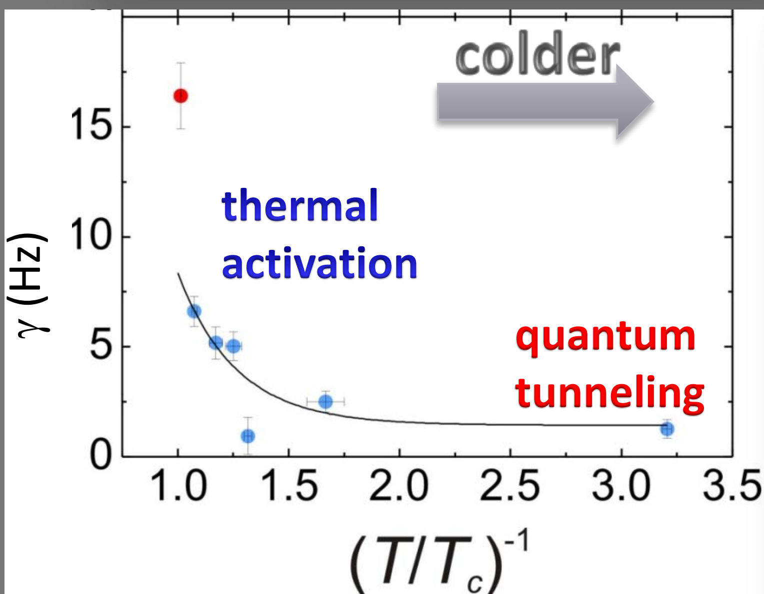
Clean system: phase slips

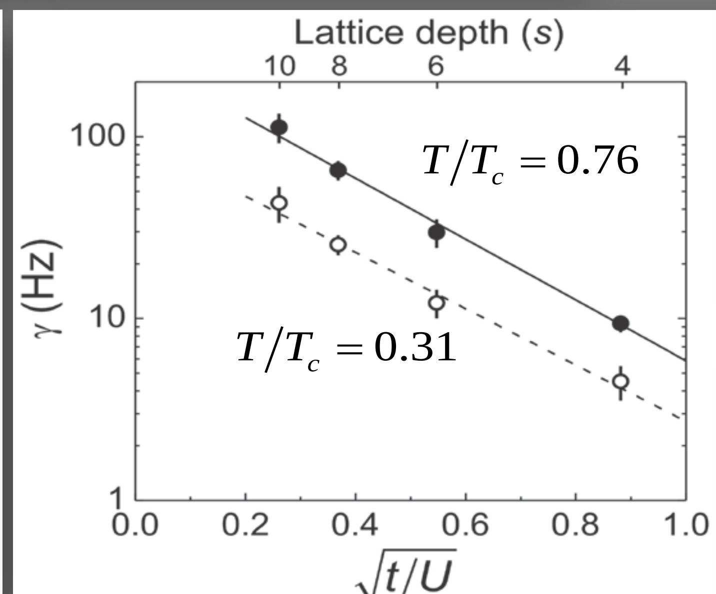
nature Vol 453 | 1 May 2008 | doi:10.1038/nature06920

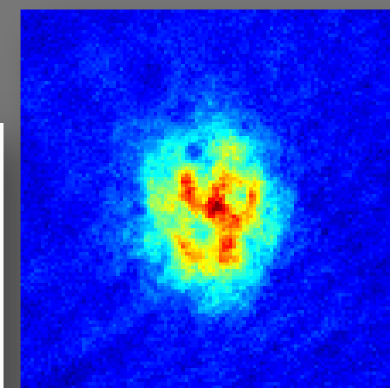
LETTERS

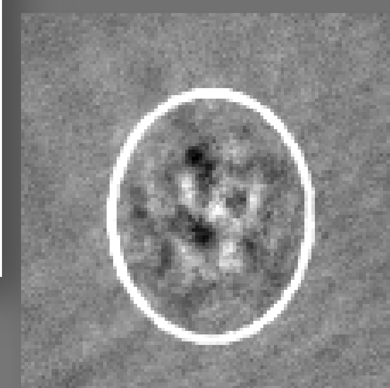
Phase-slip-induced dissipation in an atomic Bose–Hubbard system

D. McKay[1], M. White[1], M. Pasienski[1] & B. DeMarco[1]

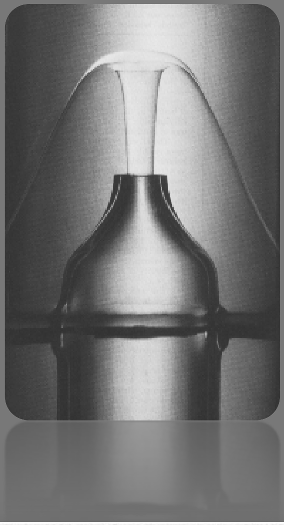




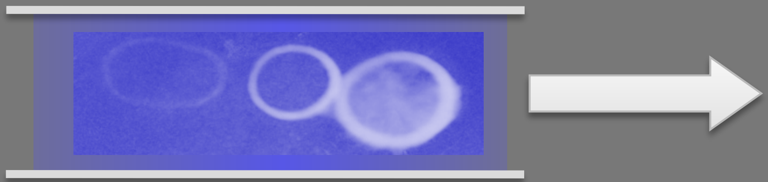


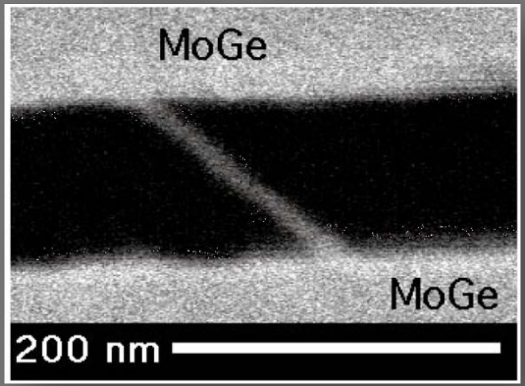


Phase slips

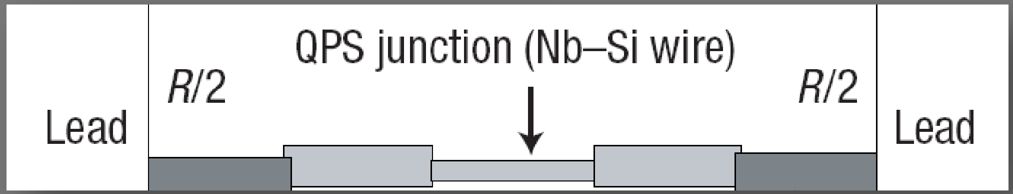


Landau got the critical velocity of SF He wrong: he didn't know about phase slips!



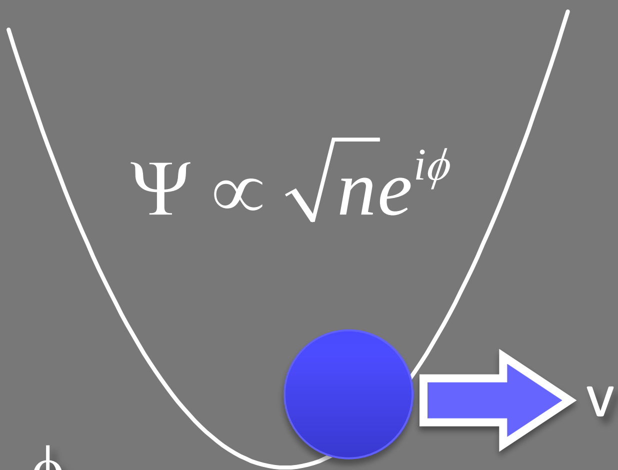


Resistance in thin superconducting wires: Quantum phase slips?



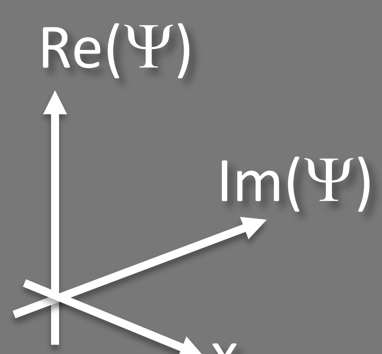
Proposed quantum phase slip current standard

Phase slips: Langer and Fisher, 1967



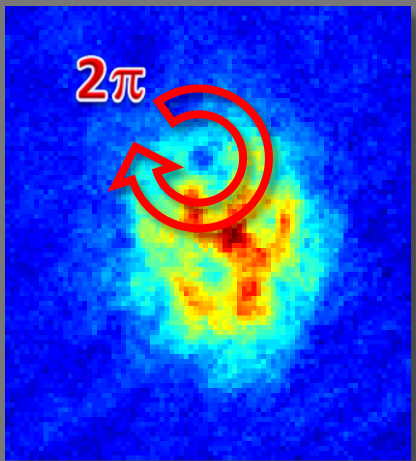
$$\Psi \propto \sqrt{n}e^{i\phi}$$

$$v \propto \frac{\partial \phi}{\partial x}$$

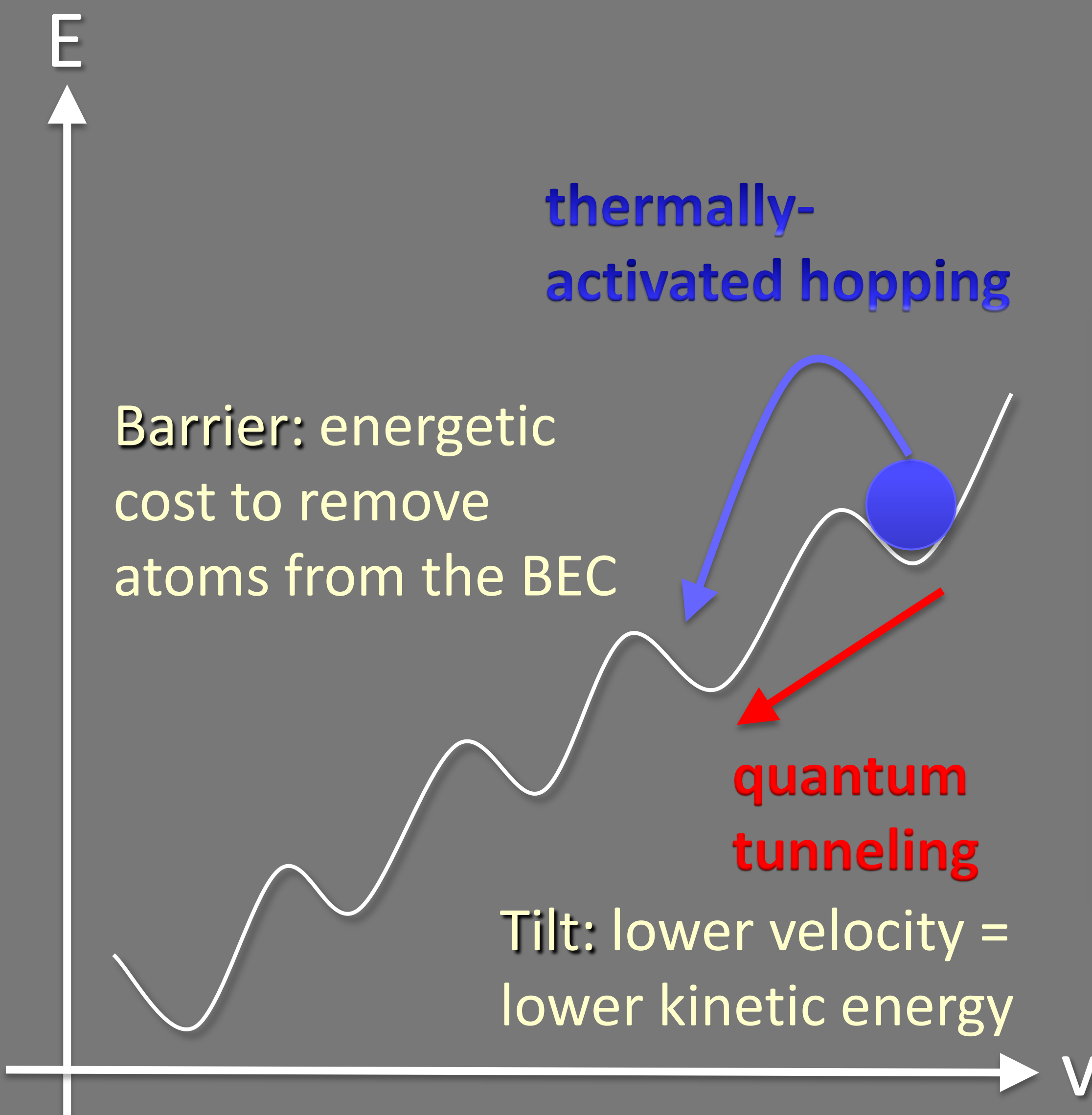


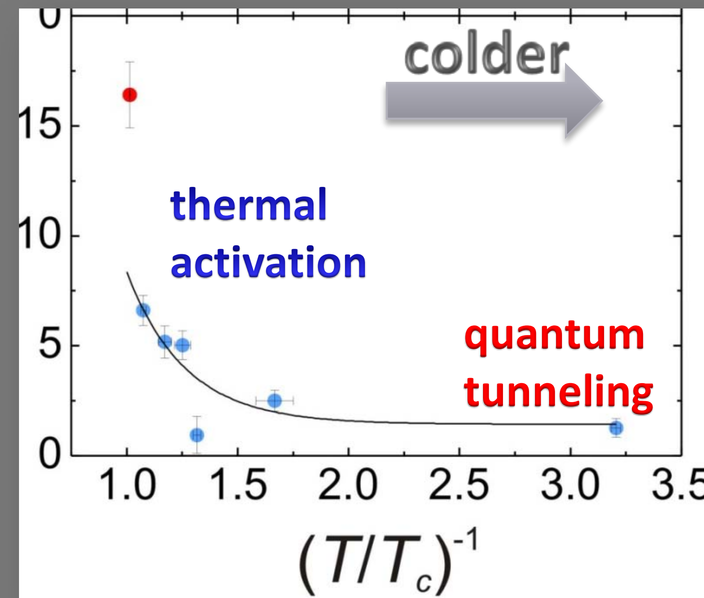
Goldbart, UIUC

What if a vortex or vortex ring nucleates and moves across the BEC?



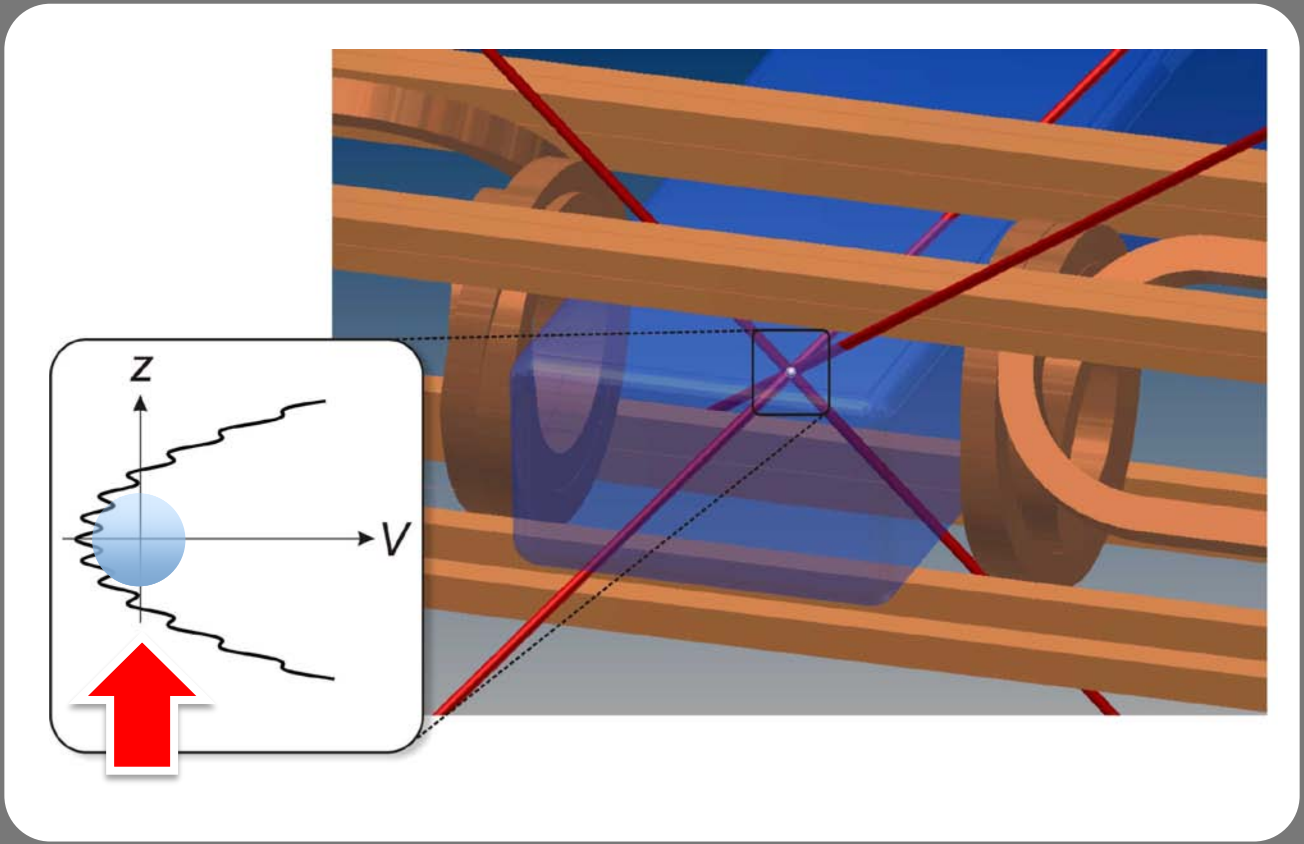
Free energy

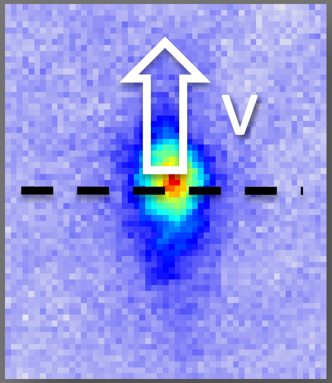

E
thermally-
activated hopping
Barrier: energetic
cost to remove
atoms from the BEC
quantum
tunneling
Tilt: lower velocity =
lower kinetic energy
V


colder
thermal
activation
quantum
tunneling
15
10
5
0
1.0
1.5
2.0
2.5
3.0
3.5
$(T/T_c)^{-1}$

Resolving an insulator

Apply impulse, measure total center-of-mass velocity



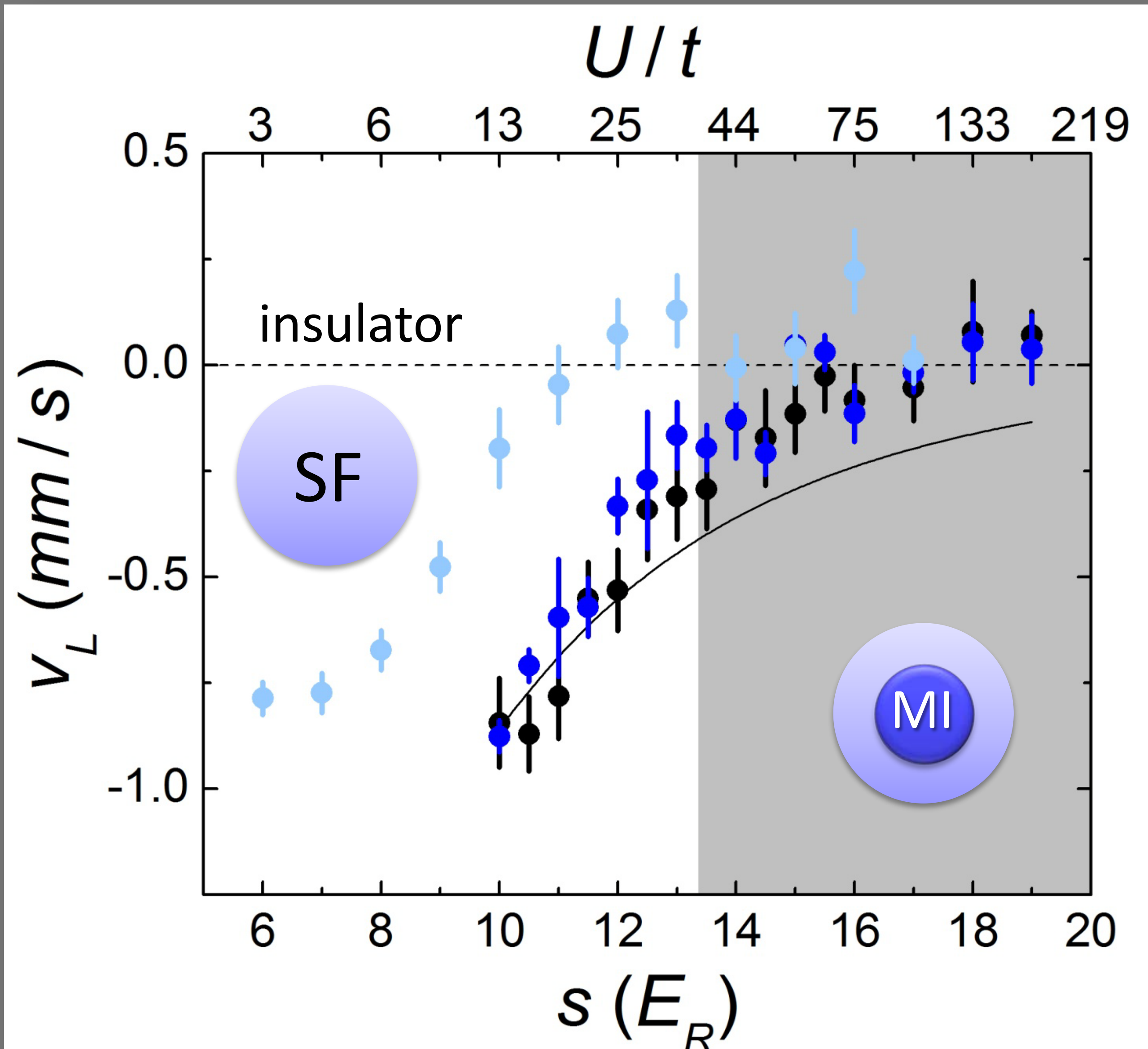


$$v_0 = \frac{N_C v_C + N_{NC} v_{NC}}{N}$$

For a damped,
SHO in the limit $\gamma \gg 1/t \gg \omega$:

$$v_0 = \frac{Ft}{m^*} e^{-\gamma t}$$

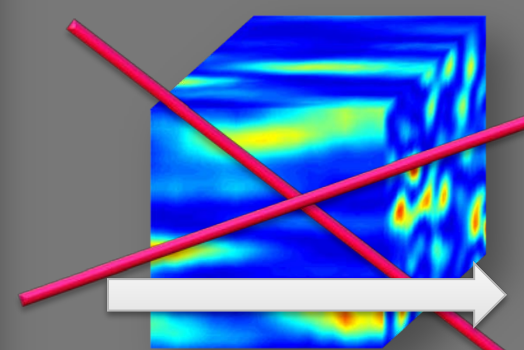
Disorder-induced SF-IN transition



Δ=3 E_R

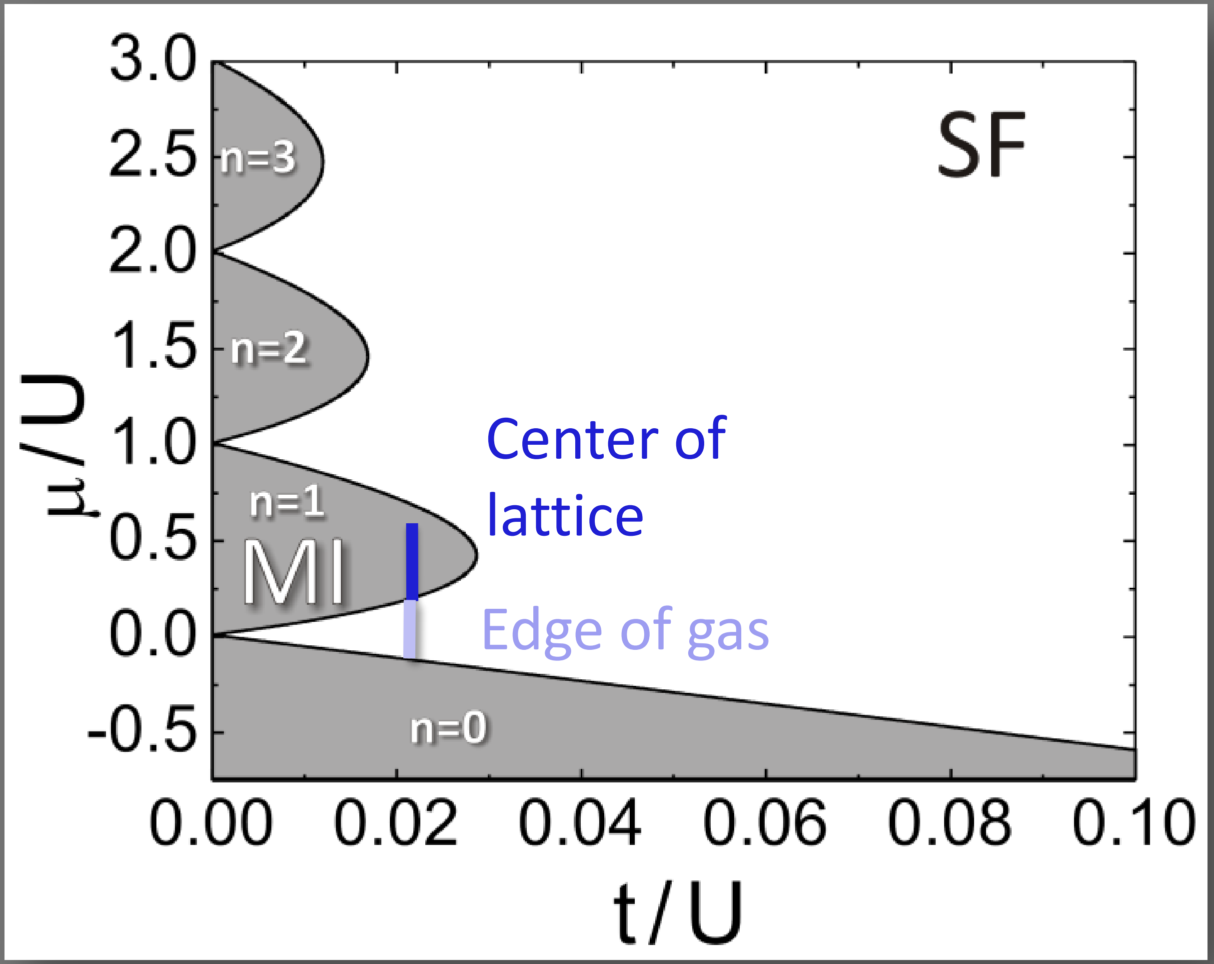
Δ=0.75 E_R

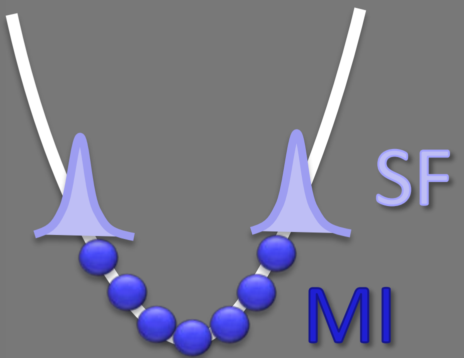
Δ=0 E_R



Co-existing phases

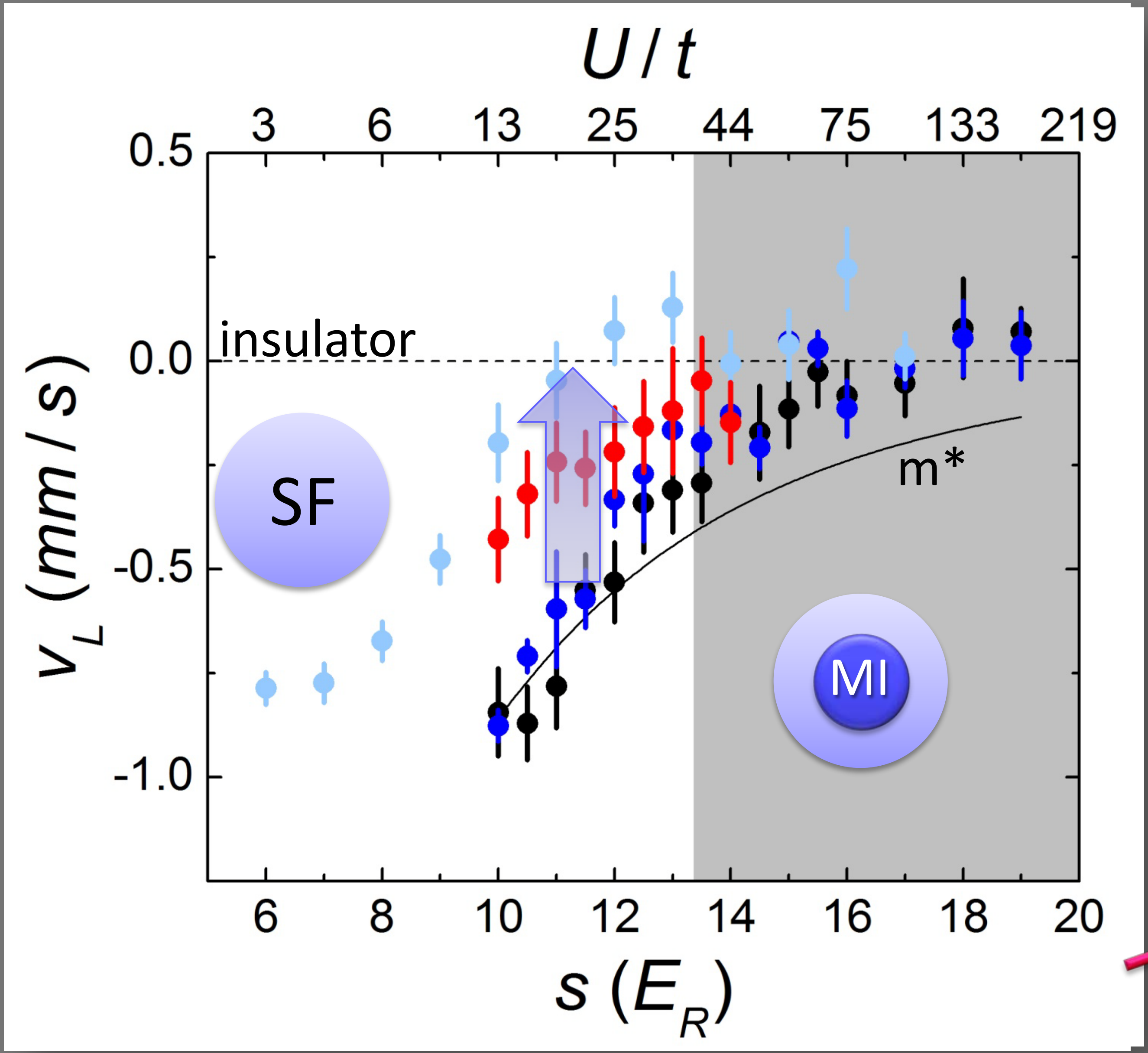
filling

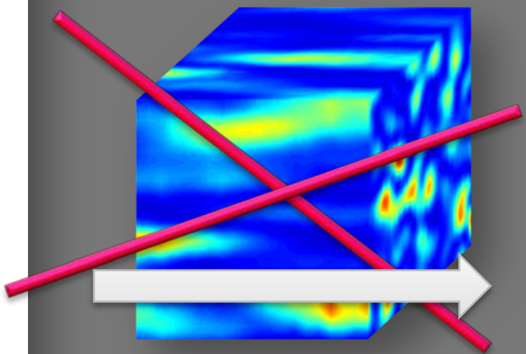




increasing lattice depth

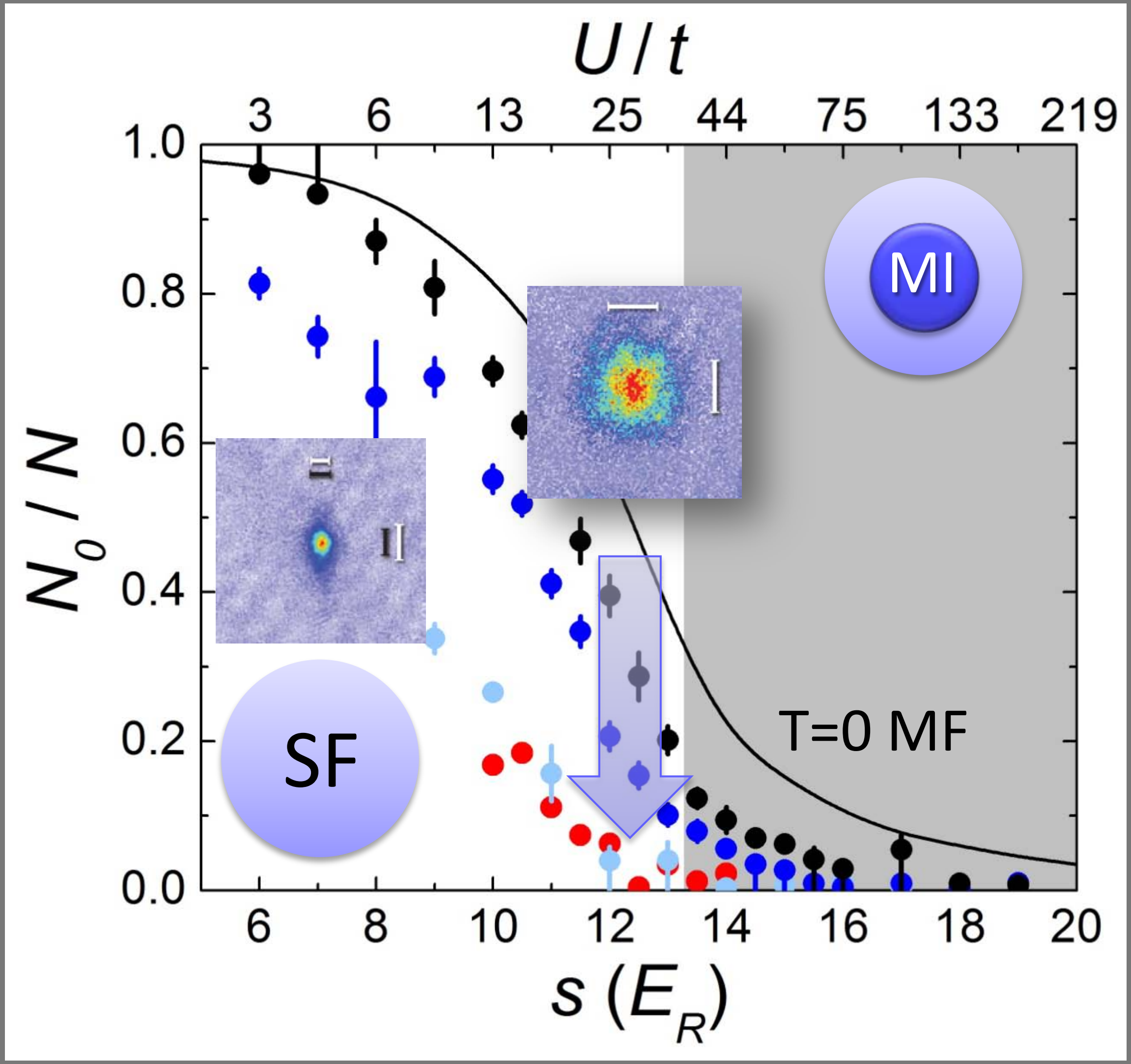
Disorder-induced SF-IN transition





Disorder-induced SF-IN transition

Destruction of condensate ➡ insulator



Δ=0 E_R

Δ=0.75 E_R

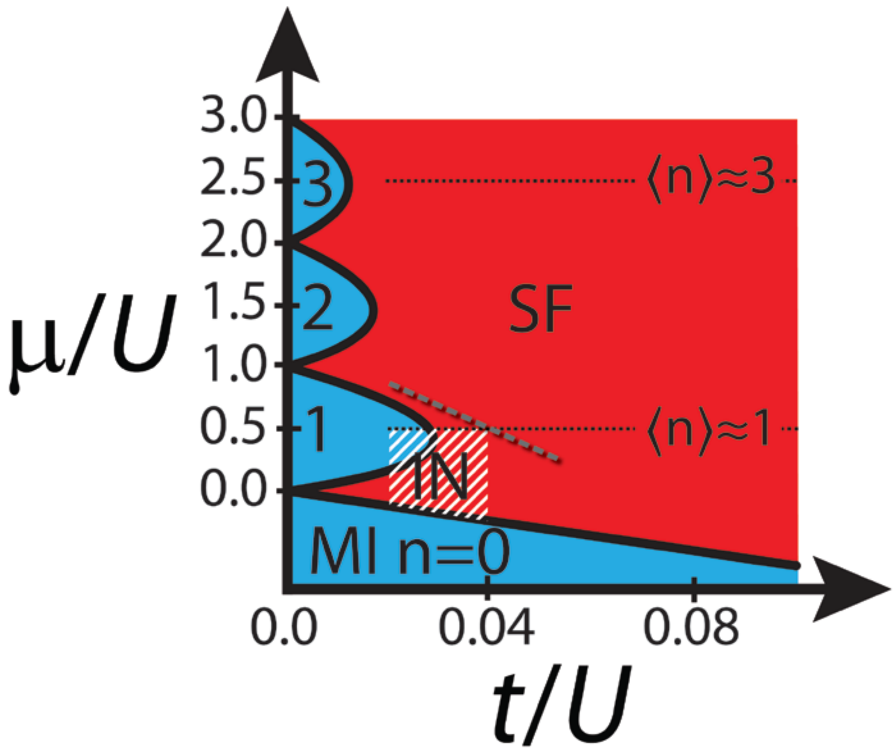
Δ=3 E_R

Δ=0 E_R hot

Conclusions

- Disorder-induced insulator for "strong" disorder
- No evidence for disorder-induced MI⇨SF transition

Nat. Phys; doi:10.1038/nphys1726 (2010)



- Temperature low enough?
- LDA?
- Finite system?
- Equilibrium?

Gurarie, Pollet, et al. arXiv:0909.4593

Bounds on entropy

Upper bound: entropy after slow (15 ms) turn off

*separate measurements indicate this is not truly adiabatic!

Lower bound: entropy before turning on lattice

Clean lattice: Estimate from N_0/N using finite temperature LDA+site-decoupled MFT